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Superconducting State in the Lagrangian Formalism of the Generalized Hubbard Model

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The nonperturbative large-N expansion applied to the generalized Hubbard model describing N-fold-degenerate correlated bands is considered. Our previous results, obtained in the framework of the Lagrangian formalism for the normal-state case, are extended to the superconducting state. The standard Feynman diagrammatics is obtained and the renormalized physical quantities are computed and analyzed. Our purpose is to obtain the $1/N$ corrections to the renormalized boson and fermion propagators when a state with Cooper-pair condensation (i.e., the superconducting state) is considered.

KEY WORDS: superconducting state; Lagrangian formalism; Hubbard model.

1. INTRODUCTION

Many problems concerning the superconductivity of strongly correlated systems were treated within the context of the generalized Hubbard model by using the decoupled slave-boson representation (Grill and Kotliar, 1990; Kotliar and Liu, 1988; Tandon *et al.*, 1999). In Grill and Kotliar (1990), Kotliar and Liu (1988), and Tandon *et al.* (1999) the generalized Hubbard model describing N-fold-degenerate correlated bands in the infinite-U limit by means of the large-N expansion was studied. Using the slave-boson technique, Fermi-liquid properties of strongly correlated systems were evaluated. Moreover, it was shown that the leading $1/N$ corrections gives rise to different superconducting instabilities depending on the band structure and the filling factor.

Since the Hubbard operator representation is quite natural to treat the electronic correlation effects (Coleman *et al.*, 2001; Izyumov, 1997), we have developed a Lagrangian formalism in which the field variables are directly the Hubbard X-operators (Foussats *et al.*, 1999, 2000, 2002). In this approach the Hubbard $\hat{X}$-operators representing the real physical excitations are treated as indivisible objects and any decoupling scheme is used.

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By using the path-integral technique, the correlation generating functional corresponding to the Lagrangian formalism was written in terms of a suitable effective Lagrangian.

Later on, in Foussats *et al.* (2002) the quantization of the t-J model in terms of the Hubbard X-operators for the normal state was given. In particular in Sections II and III of Foussats *et al.* (2002) the nonperturbative formalism for the generalized Hubbard model was analyzed. This was done by means of a new large-N expansion in the infinite-U limit carried out on our Lagrangian formalism for the t-J model. The parameter N represent the number of the electronic degrees of freedom per site and $1/N$ can be considered as a small parameter.

In this context, by defining proper propagators and vertices the standard Feynman diagrammtics of the model is given, and the bosonic and the fermionic self-energies can be renormalized. From these renormalized quantities several physical properties can be evaluated and the results were confronted with others previously obtained.

The free boson propagator which is of order $1/N$ is renormalized by series of fermionic bubbles whose contributions are also of order $1/N$.

Recently, our generalized Lagrangian model was checked by explicit computation of charge–charge and spin–spin correlation functions (Foussats and Greco, 2002). The agreement with previous results is excelent and gives a strong prove on the correcteness of our Lagrangian approach as well as of the large-N expansion.

Analogously, the fermionic self-energy up to order can be computed. Later on, from the Dyson equation the renormalized fermionic propagator can be evaluated at order $1/N$ by solving the correspondent equations self-consistently.

The self-consistent method of solution for the equations involving the dressed fermion propagator, is the usual like in the Hamiltonian formalim for the decoupled slave-bonon representation (Grilli and Kotliar, 1990; Kotliar and Liu, 1988; Tandon *et al.*, 1999).

Our model is useful to describe the normal state, i.e. the state in which the Cooper-pair amplitude $\langle c_{k\uparrow} c_{k\downarrow} \rangle$ is zero because of number conservation.

The key feature of the frequently used Bardeen–Cooper–Schriefer (BCS) theory is the Cooper-pair condensation. The simplest model that permits the description of the superconducting state is given by the BCS reduced Hamiltonian formalism. The BCS integral equation is introduced by means of Gor'kov's method. Next the superconducting state is incorporated into the formalism by using the Nambu matrix notation (Allen and Mitrovic, 1982; Nambu, 1960).

The purpose of the present paper is to make possible the description of the superconducting state in the framework of our formalism when the pair of states $(k\uparrow, -k\downarrow)$ is occupied coherently. This is done by introducing the Nambu matrix notation in the new nonperturbative large-N expansion for the generalized Hubbard model proposed in Foussats *et al.* (2002). The aim is to give the formulas for the renormalized physical quantities, such as self-energies and propagators, to leading order in $1/N$ for the superconducting state with Cooper-pair condensation.

The paper is organized as follow. In section 2, the main results of sections II and III of Foussats *et al.* (2002) are collected, and the Nambu notation is introduced. In sections 3 and 4, by using the Nambu matrix notation the Feynman diagrammatics is analyzed up to one loop with the aim to compute the $1/N$ correction to the renormalized boson propagator. The resulting expression permits us to evaluate the $1/N$ correction to the total fermion self-energy for both the normal and the superconducting states.

2. DEFINITIONS AND NAMBU NOTATION

In the slave-boson representation for the generalized Hubbard model describing N-fold-degenerate correlated bands (Grilli and Kotliar, 1990; Kotliar and Liu, 1988; Tandon *et al.*, 1999), the nonperturbative large-N expansion technique is used systematically. Also the large-N expansion was used in functional theories written in terms of the X-operators (Zeyher and Greco, 1988; Zeyher and Kulic, 1996), and was shown that in order $1/N$ the method gives different results for superconductivity.

In Foussats *et al.* (2002), by using our Lagrangian model written in the framework of the path-integral formalism a new nonperturbative large N-expansion was proposed. The generalized Hubbard model is described by means of the introduction of a set of fermion field f_{ip}, in such a way that their proportionality with the fermion-like Hubbard X_i^{0p}-operators is maintained for all order in the large-N expansion. Looking at the Lagrangian equation (2.17) of Foussats *et al.* (2002), we see that it is sufficient to retain terms up to order δR_i^2 to take into account all the terms of order $1/N$. Therefore the Lagrangian is written

$$\begin{aligned} L_{eff}^{E} = & -\frac{1}{2}\sum_{i,p}^{N}\left(\dot{f}_{ip}\, f_{ip}^{+} + \dot{f}_{ip}^{+}\, f_{ip}\right)\left(1 - \delta R_i + \delta_i^2\right) + r_0 \sum_{i,j,p}^{N} t_{ij}\, f_{ip}^{+}\, f_{ip} \\ & - (\mu' - \lambda_0)\sum_{i,p} f_{ip}^{+}\, f_{ip}(1 - \delta R_i + \delta R_i^2) \\ & + N r_0 \sum_{i} \delta\lambda_i \delta R_i + \sum_{i,p} f_{ip}^{+}\, f_{ip}\left(1 - \delta R_i + \delta R_i^2\right)\delta\lambda_i \\ & + \frac{1}{2N}\sum_{i,j,p,p'} J_{ij}[1 - (\delta R_i + \delta R_j)]\left[\, f_{ip}^{+}\, f_{ip'}\, f_{jp'}^{+}\, f_{jp} + f_{ip}^{+}\, f_{ip}\, f_{jp'}\, f_{jp'}^{+}\right]. \end{aligned} \tag{2.1}$$

As it was shown in Foussats *et al.* (2002) our Lagrangian formalism for the t-J model is a secondclass constrained system. The physical quantities such as propagators and vertices were renormalized by means of the introduction of proper ghost fields. Therefore, in the present paper we assume that all the physical quantities we must handle were previously renormalized.

Now the construction of the diagrammatics starting from the Lagrangian (2.1) in the infinite-U limit ($J_{ij} = 0$) is straightforward.

When the superconducting state is considered, the renormalized fermion propagator $\hat{\mathbf{G}}_{(\mathbf{D})}$ is *a* 2×2 matrix schematically written

$$\hat{\mathbf{G}}_{(\mathbf{D})} = \begin{pmatrix} \Rightarrow\Rightarrow & \Leftarrow\Rightarrow \\ \Rightarrow\Leftarrow & \Leftarrow\Leftarrow \end{pmatrix}, \tag{2.2}$$

where the diagonal elements with the two arrows pointing in the same direction are the normal fermionic propagators, while the nondiagonal elements with the two arrows pointing in the opposite direction are the anomalous fermionic propagators.

To describe the fermionic sector when the complete fermionic propagator is of the form (2.2), the simplest way is to introduce the Nambu matrix notation (Nambu, 1960). In this notation the two-component fermionic field operator $\boldsymbol{\Psi}_{im}(x, \tau)$ is given by

$$\Psi_{im}(x, \tau) = \begin{pmatrix} f_{im\uparrow}(x, \tau) \\ f^{+}_{im\downarrow}(x, \tau) \end{pmatrix}. \tag{2.3}$$

The Lagrangian (2.1) in terms of the field operator $\Psi_{im}(x, \tau)$ is written

$$\begin{aligned} L^E_{eff} = &-\frac{1}{2}\sum_i \sum_{m=1}^{N/2} [\dot{\Psi}^{\dagger}_{im}\hat{\mathbf{I}}\Psi_{im} - \Psi^{\dagger}_{im}\hat{\mathbf{I}}\dot{\Psi}_{im}]\left(1 - \delta R_i + \delta R_i^2\right) \\ &+ \sum_{i,j}\sum_{m=1}^{N/2} (r_0 t_{ij} - \mu\delta_{ij})\Psi^{\dagger}_{im}\hat{\tau}_3\Psi_{jm}\left(1 - \delta R_i + \delta_i^2\right) \\ &+ N r_0 \sum_i \delta\lambda_i \delta R_i + \sum_i \sum_{m=1}^{N/2} \Psi^{\dagger}_{im}\hat{\tau}_3\Psi_{jm}\left(1 - \delta R_i + \delta R_i^2\right)\delta\lambda_i, \end{aligned} \tag{2.4}$$

where we have named $\mu = \mu' - \lambda_0$.

From the Eq. (2.4) we can see that the bosonic sector described by the two field components $(\delta R_i, \delta\lambda_i)$ remains unchanged, and the fermionic sector was written by using the 2×2 Pauli matrices $\hat{\mathbf{I}}$ and $\hat{\tau}_{\mathbf{3}}$. So, to describe nondiagonal quantities appearing in the fermionic propagator when the superconducting state is considered, the four Pauli matrices are introduced

$$\hat{\mathbf{I}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2.5a}$$

$$\hat{\tau}_{\mathbf{1}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{2.5b}$$

$$\hat{\tau}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{2.5c}$$

$$\hat{\tau}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.5d}$$

The two-component fermionic field operator (2.3) in the momentum space reads

$$\Psi_{km} = \begin{pmatrix} f_{km\uparrow} \\ f^{+}_{-km\downarrow} \end{pmatrix} \tag{2.6a}$$

and

$$\Psi^{\dagger}_{km} = \begin{pmatrix} f^{+}_{km\uparrow} & f_{-km\downarrow} \end{pmatrix}. \tag{2.6b}$$

By using the above notation, in the next section we study the diagrammatics for both normal and superconducting states with the purpose to find the equation for the total fermion self-energy and hence to write the renormalized fermion propagator to leading order of large-N expansion.

3. DIAGRAMMATICS IN THE NAMBU NOTATION

The Feynman rules and diagrammatics can be obtained as is usual, and in order to compute the $1/N$ corrections to the propagators the structure of the model is examined up to one loop.

We assume the equations written in momentum space, and so once the Fourier transformation was performed, the bilinear parts of the Lagrangian (2.4) give rise to the field propagators and the remaining pieces are represented by vertices.

Moreover, as mentioned above the boson sector remains unchanged. The free boson propagator associated with the two component boson field $\delta X^a = (\delta R, \delta\lambda)$, is of order $1/N$ and is written

$$D_{(0)ab}(q, \omega_n) = \begin{pmatrix} 0 & \frac{1}{Nr_0} \\ \frac{1}{Nr_0} & 0 \end{pmatrix}, \tag{3.1}$$

where the quantities q and ω_n are respectively the momentum and the Matsubara frequency of the bosonic field.

The free propagator (3.1) is dressed by using the Dyson equation $(D_{ab})^{-1} = (D_{(0)ab})^{-1} - \prod_{ab}^{(\mathrm{Ren})}$. The boson self-energy and the dressed components $D_{RR}(q, \omega_n)$, $D_{\lambda R}(q, \omega_n)$, and $D_{\lambda\lambda}(q, \omega_n)$ of the matricial boson propagator were found in Foussats *et al.* (2002), Eqs. (4.4) and (4.5), respectively.

The renormalized boson propagator we found is the suitable one that permits us to evaluate for instance the $1/N$ correction to the fermion self-energy.

Finally we remark that when only the normal state is considered, our diagrammatics was checked by computing numerically the charge–charge and spin–spin correlation functions on the square lattice for nearest-neighbor hopping t (Foussats and Greco, 2002). The results are in agreement with previous ones arising from the slave-boson model as well as from the functional X-operators canonical approach (Gehlhoff and Zeyher, 1965; Wang, 1992).

Now we briefly analyze the fermionic sector of the Lagrangian (2.4) for the normal state.

The bilinear fermionic part of the Lagrangian (2.4) in the momentum space reads

$$L^F(\Psi^\dagger_{km}, \Psi_{km}) = -\frac{1}{2}\sum_k \sum_{m=1}^{N/2} \Psi^\dagger_{km}(\hat{\mathbf{G}}_{(\mathbf{0})})^{-1}\Psi_{km}, \tag{3.2}$$

where the 2×2 matrix $(\hat{\mathbf{G}}_{(\mathbf{0})})^{-1}$ is given by

$$(\hat{\mathbf{G}}_{(\mathbf{0})})^{-1} = -[i\nu_n\hat{\mathbf{I}} - (\varepsilon_k - \mu)\hat{\tau}_3] = -[i\nu_n\hat{\mathbf{I}} - \Delta_k\hat{\tau}_3] \tag{3.3}$$

and whose determinant writes

$$\det(\hat{\mathbf{G}}_{(\mathbf{0})})^{-1} = -[(i\nu_n)^2 - (\Delta_k)^2] = \nu_n^2 + \Delta_k^2, \tag{3.4}$$

where was defined $\varepsilon_k = -r_0 t \sum_I \exp(-iI \cdot k)$; and I is the lattice vector. The quantities k and ν_n are respectively the momentum and the Matsubara frequency of the fermionic field.

Therefore, the free fermion propagator $\hat{\mathbf{G}}_{(\mathbf{0})}$ is

$$\hat{\mathbf{G}}_{(\mathbf{0})}(k, i\nu_n) = -\begin{pmatrix} \frac{1}{i\nu_n - \Delta_k} & 0 \\ 0 & \frac{1}{i\nu_n + \Delta_k} \end{pmatrix}, \tag{3.5}$$

where we call $\Delta_k = (\varepsilon_k - \mu)$, having the property $\Delta_k = \Delta_{-k}$. From this property it can be seen that

$$G_{(\mathbf{0})22}(k, i\nu_n) = -G_{(\mathbf{0})11}(-k, -i\nu_n). \tag{3.6}$$

For noninteracting band electrons, the off-diagonal elements in (2.2) vanish, and the element $G_{(\mathbf{0})11}$ has the usual scalar form $-(i\nu_n - \Delta_k)^{-1}$ (see Foussats *et al.*, 2002).

The matrix equation (3.5) in terms of the Pauli matrices can be written

$$\begin{aligned} \hat{\mathbf{G}}_{(\mathbf{0})}(k, i\nu_n) &= -(i\nu_n\hat{\mathbf{I}} - \Delta_k\hat{\tau}_{\mathbf{3}})^{-1} = -\frac{(i\nu_n\hat{\mathbf{I}} + \Delta_k\hat{\tau}_{\mathbf{3}})}{(i\nu_n)^2 - \Delta_k^2} \\ &= \frac{1}{\det(\hat{\mathbf{G}}_{(\mathbf{0})})^{-1}}(i\nu_n\hat{\mathbf{I}} + \Delta_k\hat{\tau}_{\mathbf{3}}). \end{aligned} \tag{3.7}$$

Looking at the Eq. (2.4) it can be seen that three-leg (one boson and two fermions) and four-leg (two bosons and two fermions) vertices, are respectively originated by the following pieces of that Lagrangian

$$
\begin{aligned}
L^{B,2F}(\delta X^a, \Psi^\dagger_{im}, \Psi_{im}) = \frac{1}{2}\sum_i \sum_{m=1}^{N/2} [\dot{\Psi}^\dagger_{im}\hat{\mathbf{I}}\Psi_{im} - \Psi^\dagger_{im}\hat{\mathbf{I}}\dot{\Psi}_{im}]\,\delta R \\
+ \mu \sum_i \sum_{m=1}^{N/2} \Psi^\dagger_{im}\hat{\tau}_3\Psi_{jm}\,\delta R \\
+ \sum_i \sum_{m=1}^{N/2} \Psi^\dagger_{im}\hat{\tau}_3\Psi_{jm}\,\delta\lambda,
\end{aligned}
\tag{3.8}
$$

$$
\begin{aligned}
L^{2B,2F}(\delta X^a, \delta X^b, \Psi^\dagger_{im}, \Psi_{im}) = -\frac{1}{2}\sum_i \sum_{m=1}^{N/2} [\dot{\Psi}^\dagger_{im}\hat{\mathbf{I}}\Psi_{im} - \Psi^\dagger_{im}\hat{\mathbf{I}}\dot{\Psi}_{im}]\,\delta R^2 \\
- \mu \sum_i \sum_{m=1}^{N/2} \Psi^\dagger_{im}\hat{\tau}_3\Psi_{jm}\,\delta R^2 \\
- \sum_i \sum_{m=1}^{N/2} \Psi^\dagger_{im}\hat{\tau}_3\Psi_{jm}\,\delta R\delta\lambda.
\end{aligned}
\tag{3.9}
$$

Therefore, the vertices can be written

$$
\Lambda^{mm'}_a = (-1)\left[\frac{i}{2}(\nu_n + \nu'_n)\hat{\mathbf{I}} + \mu\hat{\tau}_3,\ \hat{\tau}_3\right]\delta^{mm'}, \tag{3.10}
$$

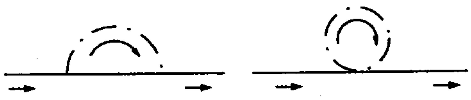
$$
\Lambda^{mm'}_{ab} = \frac{1}{2}\begin{pmatrix} i(\nu_n + \nu'_n)\hat{\mathbf{I}} + \mu\hat{\tau}_3 & \hat{\tau}_3 \\ \hat{\tau}_3 & 0 \end{pmatrix}\delta^{mm'}. \tag{3.11}
$$

From the above Feynman diagrammatics the expression for the $1/N$ correction to the fermion self-energy for the normal and the superconducting states can be written.

4. THE 1/*N* CORRECTION TO THE FERMION SELF-ENERGY FOR THE NORMAL AND THE SUPERCONDUCTING STATES

As it was commented in the Introduction the simplest model suitable to describe the superconducting state is given by the BCS reduced Hamiltonian formalism. In the normal state such formalism reduces to Migdal's theory whose essence is to use only the lowest order Feynman diagram provided by the reduced Hamiltonian (Bardeen *et al.*, 1957).

In our model the total fermion self-energy $\sum$ for the normal state is given by the sum of contributions corresponding to the following two one-loop diagrams



$$\hat{\Sigma} = \hat{\Sigma}^{(1)} + \hat{\Sigma}^{(2)} \tag{4.1}$$

In the Nambu matrix notation the matrices $\hat{\Sigma}^{(1)}$ and $\hat{\Sigma}^{(2)}$ respectively are written

$$\begin{aligned}\hat{\Sigma}^{(1)}(k, i\nu_n) &= \frac{1}{N_s} \sum_{m,m',q,\omega_n} \Lambda_a^{mm'} D_{(V)}^{ab}(q, \omega_n) \Lambda_b^{m'm} \hat{\mathbf{G}}_{(\mathbf{0})}(\nu_n + \omega_n, k + q) \\ &= \frac{1}{N_s} \sum_{q,\omega_n} \left\{ \left[-\frac{1}{4}(2\nu_n + \omega_n)^2 + \eta^2 \hat{\mathbf{I}} + i(2\nu_n + \omega_n)\mu\hat{\tau}_3 \right] D_{(V)}^{RR}(q, \omega_n) \right. \\ &\quad + 2\left[\frac{i}{2}(2\nu_n + \omega_n)\hat{\tau}_\mathbf{3} + \mu\hat{\mathbf{I}} \right] D_{(V)}^{R\lambda}(q, \omega_n) \\ &\quad \left. + \hat{\mathbf{I}} D_{(V)}^{\lambda\lambda}(q, \omega_n) \right\} \hat{\mathbf{G}}_{(0)}(\nu_n + \omega_n, k + q), \end{aligned} \tag{4.2}$$

$$\begin{aligned}\hat{\Sigma}^{(2)}(k, i\nu_n) &= \sum_{m,m',q,\omega_n} \Lambda_{ab}^{mm'} D_{(V)}^{ab}(q, \omega_n) \\ &= \sum_{q,\omega_n} \frac{1}{2}[i(2\nu_n + \omega_n)\hat{\mathbf{I}} + \mu\hat{\tau}_3] D_{(V)}^{RR}(q, \omega_n) + \hat{\tau}_3 \sum_{q,\omega_n} D_{(V)}^{R\lambda}(q, \omega_n). \end{aligned} \tag{4.3}$$

From the above equations the $1/N$ correction to the fermion self-energy can be computed (see, for instance, (Kotlian and Liu, 1988)).

Alternatively the matrices $\hat{\Sigma}^{(1)}$ and $\hat{\Sigma}^{(2)}$ can be written

$$\hat{\Sigma}^{(1)}(k, i\nu_n) = \begin{pmatrix} A_1 + B_1 & 0 \\ 0 & A_1 - B_1 \end{pmatrix} \hat{\mathbf{G}}_{(\mathbf{0})}(\nu_n + \omega_n, k + q), \tag{4.4}$$

$$\hat{\Sigma}^{(2)}(k, i\nu_n) = \begin{pmatrix} A_2 + B_2 & 0 \\ 0 & A_2 - B_2 \end{pmatrix}, \tag{4.5}$$

where

$$\begin{aligned} A_1 = \sum_{q,\omega} \Bigg[&\left(-\frac{1}{4}(2\nu_n + \omega_n)^2 + \mu^2 \right) D_{(V)}^{RR}(q, \omega_n) \\ &+ \mu D_{(V)}^{R\lambda}(q, \omega_n) + D_{(V)}^{\lambda\lambda}(q, \omega_n) \Bigg], \end{aligned} \tag{4.6a}$$

$$B_1 = \mu \sum_{q,\omega} i(2\nu_n + \omega_n)\left[D^{RR}_{(V)}(q,\omega_n) + D^{R\lambda}_{(V)}(q,\omega_n)\right], \tag{4.6b}$$

$$A_2 = \sum_{q,\omega} \frac{i}{2}(2\nu_n + \omega_n) D^{RR}_{(V)}(q,\omega_n), \tag{4.6c}$$

$$B_2 = \sum_{q,\omega} \frac{1}{2}\mu D^{RR}_{(V)}(q,\omega_n) + D^{R\lambda}_{(V)}(q,\omega_n). \tag{4.6d}$$

Therefore, for the normal state the total fermion self-energy is a diagonal matrix which in terms of the Pauli matrices can be explicitly written as follows:

$$\hat{\Sigma}(k, i\nu_n) = -i\nu_n[1 - Z(k, i\nu_n)]\hat{\mathbf{I}} + \chi(k, i\nu_n)\hat{\tau}_{\mathbf{3}}. \tag{4.7}$$

The fermionic dressed propagator is defined by means of the Dyson equation $(\hat{\mathbf{G}}_{(\mathbf{D})})^{-1}(k, i\nu_n) = (\hat{\mathbf{G}}_{(\mathbf{0})})^{-1}(k, i\nu_n) - \hat{\Sigma}(k, i\nu_n)$.

The above equations are suitable to describe the leading $1/N$ corrections for the normal state in the generalized Hubbard model describing N-fold-degenerate correlated bands in the infinite-U limit. They were obtained by means of a new nonperturbative large-N expansion in the framework of our Lagrangian model.

Now the superconducting state must be incorporated. By looking at the expression of the fermionic self-energy (Eq. (4.7)) we assume that the most general form to write the total self-energy in terms of the Pauli matrices is

$$\hat{\Sigma}(k, i\nu_n) = -i\nu_n[1 - Z(k, i\nu_n)]\hat{\mathbf{I}} + \chi(k, i\nu_n)\hat{\tau}_3 + \phi(k, i\nu_n)\hat{\tau}_{\mathbf{1}} + \overline{\phi}(k, i\nu_n)\hat{\tau}_2, \tag{4.8}$$

where Z, χ, ϕ, and $\overline{\phi}$ are four independent arbitrary functions.

When the superconducting state is taken into account the "anomalous" dressed fermionic propagator also is determined by the Dyson equation, consequently

$$\left(\hat{\mathbf{G}}_{(\mathbf{D})}\right)^{-1}(k, i\nu_n) = -i\nu_n Z\hat{\mathbf{I}} - (\chi - \Delta_k)\hat{\tau}_{\mathbf{3}} - \phi(k, i\nu_n)\hat{\tau}_1 - \overline{\phi}(k, i\nu_n)\hat{\tau}_2. \tag{4.9}$$

This matrix can be inverted, and it results

$$\begin{aligned}\hat{\mathbf{G}}_{(\mathbf{D})}(k, i\nu_n) = \frac{1}{\det\left(\hat{\mathbf{G}}_{(\mathbf{D})}\right)^{-1}}&[-i\nu_n Z\hat{\mathrm{I}} + (\chi - \Delta_k)\hat{\tau}_{\mathbf{3}} - \phi(k, i\nu_n)\hat{\tau}_{\mathbf{1}} \\ &+ \overline{\phi}(k, i\nu_n)\hat{\tau}_{\mathbf{2}}],\end{aligned} \tag{4.10}$$

where

$$\det\left(\hat{\mathbf{G}}_{(\mathbf{D})}\right)^{-1}(k, i\nu_n) = (i\nu_n Z)^2 - (\chi - \Delta_k)^2 - \phi^2 - \overline{\phi}^2. \tag{4.11}$$

It is clear that the Dyson perturbation series for the matrix $\hat{\mathbf{G}}_{(\mathbf{D})}$ turns out to be identical to that for $G_{(D)}$. The only difference is that $\hat{\mathbf{G}}_{(\mathbf{D})}$ is a matrix and that factors of the Pauli matrices are attached in the interaction matrix elements.

Since in the normal state the fermionic propagator $\hat{\mathbf{G}}_{(\mathbf{D})}$ is diagonal, it is clear that the both arbitrary functions ϕ and $\overline{\phi}$ must vanish. The arbitrary functions Z and χ are univocally determined by the normal state, and in order to verify the property (3.6) both quantities must be even functions of $i\nu_n$. The "normal" solution $\phi = \overline{\phi} = 0$ always exists. So, the functions Z and χ in the normal state remain defined by the following equations

$$i\nu_n[1 - Z(k, i\nu_n)] = \frac{1}{2}\big[\Sigma(k, i\nu_n) - \Sigma(k, -\nu_n)\big], \tag{4.12}$$

$$\chi(k, i\nu_n) = \frac{1}{2}\big[\Sigma(k, i\nu_n) + \Sigma(k, -i\nu_n)\big], \tag{4.13}$$

where it was assumed that everything is even in the momentum k.

Moreover, it is assumed that the property (3.6) is mantained in the superconducting state, and so is necessary that $\phi^2 + \overline{\phi}^2$ must be also an even function in $i\nu_n$. Also, it is possible to see that ϕ and $\overline{\phi}$ satisfy identical nonlinear equations. Consequently, except a proporcionality factor (phase factor) both functions must be equals. When a solution $(\phi, \overline{\phi})$ with one or both functions different from zero exists, it is possible to show that it describes the state with Cooper-pair condensation (the superconducting state) (Bardeen and Stephen, 1964). The simplest solution is to take $\phi \neq 0$ and $\overline{\phi} = 0$ corresponding to fix the phase factor. This is possible because the physical observables cannot depend of this phase. This choice is equivalent to write the self-energy in terms of the real Pauli matrices.

Finally, as it occurs in the normal-state case, the equation for the total fermionic self-energy must be solved self-consistently by using the Eq. (4.10). As it is usual the explicit computation is carried out by introducing the spectral representation of the boson propagator.

5. CONCLUSIONS

As commented above nowadays the BCS theory is the remarkably model capable to describe the superconducting state. The main feature of BCS theory is the Cooper-pair condensation, in this approach the pair of states $(k \uparrow, -k \downarrow)$ is occupied coherently. The Cooper-pair amplitude $\langle C_{k\uparrow}\, c_{-k\downarrow}\rangle$ which is zero in the normal-state due to the number conservation becomes finite bellow T_{c}. The simplest model which permits such behavior is the BCS reduced Hamiltonian model.

Recently, a Lagrangian family that can be mapped in the slave-boson representation was studied (Foussats *et al.*, 2002). In the case of the normal-state the nonperturbative formalism for the generalized Hubbard model by using a new large-N expansion in the infinite-U limit was given. The standard Feynman diagrammatics was constructed, in order to compute the $1/N$ correction to the boson propagator. The structure of the model was examined in detail up to one loop. The renormalized boson propagator we found is the suitable one that permits us to evaluate the $1/N$ correction to the fermion self-energy. In the normal-state case, the diagrammatics was checked by computing numerically the charge–charge and spin–spin correlation functions on the square lattice for nearest-neighbor hopping t (Foussats and Greco, 2002). The results obtained in Foussats and Greco (2002) are in agreement with previous one arising from the slave-boson model as well as from the functional X-operators canonical approach.

In this paper, by using the Nambu matrix notation we have rewritten the Lagrangian for the t-J model and the Feynman diagrammatics was constructed but now taking into account the superconducting state. In this situation propagators and vertices were again evaluated. The renormalized physical quantities to leading order in $1/N$ were computed, and the equation for the total fermion self-energy which must be solved self-consistently was found. So, we have given the theoretical framework suitable to describe the superconducting state in the Lagrangian formalism for the generalized Hubbard model. In a future work our equations will be checked by computing the relevant physical quantities. Also the more general case with $j_{ij} \neq 0$ which incorporate the four-leg fermion vertex will be studied.

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